Causal Propagators for the Second Order Wilson Loop

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Abstract

We evaluate the Wilson loop at second order in general non-covariant gauges by means of the causal principal-value prescription for the gauge-dependent poles in the gauge-boson propagator and show that the result agrees with the usual causal prescriptions.

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Its well known that the QCD vacuum structure is particularly simple in the light-front formalism; for instance, the infinity-momentum frame, which fits in this formalism, is appropriate to the study of deeply-inelastic scattering processes. The rich underlying phenomenology has raised a lot of interest in the study of non-Abelian gauge theories in general axial-type gauges, despite some mathematical shortcomings that, fortunately, can be controled by physical requirements, where causality plays the most important role.

In the context of the gauge-invariant dynamics of QCD, the Wilson loop is certainly one of the most relevant objects in such theories, from the conceptual standpoint. In 1982, the computation of the Wilson loop to the fourth order carried out by Caracciolo *et al*[1] has revealed that in the temporal gauge the naïve Cauchy principal-value (PV) prescription used to handle the gauge-dependent poles in the gauge-boson propagator leads to results which

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fails to agree with the ones obtained in the Feynman and Coulomb gauges. Later, Basseto *et al*[2] arrived at the correct result using the unified-gauge formalism introduced by Leibbrandt[3].

In 1991, Pimentel and Suzuki[4] have proposed a causal principal-value prescription for the light-cone gauge by conjecturing that the gauge-boson propagator as a whole must be causal[5]. Recently causal propagators for non-covariant gauges were derived on the same grounds as for covariant radial distributions[6] and shown to coincide with the resulting distributions obtained through the causal prescription of Pimentel and Suzuki, except for the pure axial gauge[7].

In the present work we test the consistency of the Pimentel-Suzuki prescription by calculating the Wilson loop at one-loop order following the manifestly gauge-invariant procedure of Hand and Leibbrandt[8], who have used distinct sets of vectors n_μ, n_μ^* and N_μ, N_μ^* for the paths and gauge-fixing constraint, respectively.

The Lagrangian density for the massless Yang-Mills theory is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{2\alpha}(N.A^a)(N.A_a) \ , \ \alpha \to 0 \ , \tag{1}$$

where $N_\mu = (N_0, \mathbf{N})$ is the gauge-fixing vector, and

$$N^\mu A_\mu^a = 0 \ , \ \mu = 0, \ldots 3 \ , \tag{2}$$

the gauge-fixing constraint.

The one-loop expectation value of the Wilson loop for a rectangular path lying in Minkowski space, characterized in terms of two light-cone vectors $n_\mu = (n_0, \mathbf{n})$ and $n_\mu^* = (n_0, -\mathbf{n})$, is

$$W^{(1)} = (ig^2)C_F\mu^{4-D} \int \frac{d^Dk}{(2\pi)^D} G_{\mu\nu}(k) \int_0^1 dt \int_0^1 dt' [n_\mu^* n_\nu^* F_1(t,t')$$
$$+n_\mu n_\nu F_2(t,t') + n_\mu n_\nu^* F_3(t,t')] \ , \tag{3}$$

where

$$F_1(t,t') \equiv e^{ik.n^*(t-t')} - e^{-ik.n^*(t-t')+ik.n} \ , \tag{4}$$
$$F_2(t,t') \equiv e^{ik.n(t-t')} - e^{-ik.n(t-t')-ik.n^*} \ , \tag{5}$$
$$F_3(t,t') \equiv e^{-ik.n^*t+ik.nt'+ik.n^*} - e^{-ik.nt+ik.n^*t'+ik.(n-n^*)}$$
$$+e^{-ik.n^*t+ik.nt'-ik.n} - e^{ik.n^*t-ik.nt'} \ , \tag{6}$$

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and
$$G_{\mu\nu}(k) = \frac{-i}{k^2 + i\epsilon} \left[g_{\mu\nu} - \frac{k_\mu N_\nu + k_\nu N_\mu}{k.N} + \frac{N^2}{(k.N)^2} k_\mu k_\nu \right] \tag{7}$$

is the gluon propagator.

Performing the integration over the path variables t and t' we obtain after contraction of the Lorentz indices

$$W^{(1)} = (ig)^2 C_F \mu^{4-D} 2n.n^* \int \frac{d^D k}{(2\pi)^D} \frac{-i}{k^2 + i\epsilon} \frac{1}{(k.n)(k.n^*)}$$
$$\times [-2 + 2e^{ik.n} + 2e^{ik.n^*} - e^{ik.(n+n^*)} - e^{ik.(n-n^*)}] \ . \tag{8}$$

Instead of choosing any *ad hoc* prescription to treat the poles $(k.N)^{-1}$ and $(k.N)^{-2}$ in (7) we now apply the principle of analytic continuation in order to derive the causal distribution corresponding to the gauge-boson propagator. For this purpose let us consider the product $[k^2(k.n)^m]^{-1}$ with $n^\mu \equiv (n^0, 0, 0, n^3)$ being an external arbitrary vector. The factor $[k^2(k.n)^m]^{-1}$ upon the hypothesis of analytic continuation to the upper complex half-plane becomes

$$\frac{1}{k^2(k.n)^m} \rightarrow \frac{1}{(k^2 + 2i\epsilon k_0^2)(k.n + i\epsilon k^0 n^0)^m}. \tag{9}$$

Due to the arbitrariness of n, it can be chosen so that $n^0 > 0$, and since ϵ is strictly positive, equation (9) becomes

$$\frac{1}{k^2(k.n)^m} \rightarrow \frac{1}{(k^2 + 2i\epsilon k_0^2)(k.n + i\epsilon|k^0|n^0)^m}, \quad \text{for } k^0 > 0$$

$$\frac{1}{k^2(k.n)^m} \rightarrow \frac{1}{(k^2 + 2i\epsilon k_0^2)(k.n - i\epsilon|k^0|n^0)^m}, \quad \text{for } k^0 < 0 \tag{10}$$

or, using the Heaviside distribution,

$$\frac{1}{k^2(k.n)^m} \rightarrow \frac{1}{k^2 + i\varepsilon} \left\{ \frac{\Theta(-k^0)}{(k.n - i\xi)^m} + \frac{\Theta(k^0)}{(k.n + i\xi)^m} \right\}, \quad \begin{matrix} \varepsilon \equiv 2\epsilon k_0^2 \rightarrow 0^+ \\ \xi \equiv \epsilon|k^0|n^0 \rightarrow 0^+ \end{matrix}, \tag{11}$$

which is just the causal prescription considered in reference [2] for m=2. We can extend the above derivation to the case $n^0 = 0$ if, before analytic continuing k^0, we first perform an infinitesimal Lorentz transformation

$$n^\mu \rightarrow n'^\mu = \Lambda^\mu{}_\nu n^\nu = \Lambda^\mu{}_3 n^3 \ , \quad \Lambda^0{}_3 n^3 > 0 \ , \tag{12}$$

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and then return to the original Lorentz frame, so that the gauge condition (2) is preserved.

Thus, making the substitution (11) in equation (8) for m=1 in order to treat the pole $(k.n)^{-1}$, and using the distribution identity

$$\frac{1}{k.n \pm i\xi} = PV\frac{1}{k.n} \mp i\pi\delta(k.n) \ , \tag{13}$$

we arrive at

$$W^{(1)} = (ig^2)C_F\mu^{4-D}2n.n^*(I_{PV} + I_\delta) \ , \tag{14}$$

where we have defined

$$I_{PV} \equiv \frac{1}{2}\int\frac{d^Dk}{(2\pi)^D}\frac{-i}{k^2 + i\epsilon}\left\{\frac{1}{k.n + i\xi} + \frac{1}{k.n - i\xi}\right\}\frac{1}{k.n^*}$$
$$\times[-2 + 2e^{ik.n} + 2e^{ik.n^*} - e^{ik.(n+n^*)} - e^{ik.(n-n^*)}]$$
$$= \frac{1}{2}\int\frac{d^Dk}{(2\pi)^D}\frac{-i}{k^2 + i\epsilon}\left\{\frac{1}{(k.n)(k.n^*) + i\xi} + \frac{1}{(k.n)(k.n^*) - i\xi}\right\}$$
$$\times[-2 + 2e^{ik.n} + 2e^{ik.n^*} - e^{ik.(n+n^*)} - e^{ik.(n-n^*)}] \ , \tag{15}$$

$$I_\delta \equiv -i\pi\int\frac{d^Dk}{(2\pi)^D}\frac{-i}{k^2 + i\epsilon}\frac{1}{k.n^*}\varepsilon(k^0)\delta(k.n)$$
$$\times[-2 + 2e^{ik.n} + 2e^{ik.n^*} - e^{ik.(n+n^*)} - e^{ik.(n-n^*)}] \ . \tag{16}$$

Performing the integral over k_0, we see that I_δ vanishes by symmetric integration:

$$I_\delta = -i\pi n_0\int\frac{d^{D-1}\mathbf{k}}{(2\pi)^D}\frac{1}{[(\mathbf{k}.\mathbf{n})^2 - n_0^2\mathbf{k}^2]}\frac{\sin(2\mathbf{k}.\mathbf{n})}{|\mathbf{k}.\mathbf{n}|} = 0 \ , \tag{17}$$

since the integrand is an odd function in the components of the vector $\mathbf{k}$. On the other hand, making use of identity (13) once more, we rewrite the remaining integral I_{PV} in the form

$$I_{PV} = \int\frac{d^Dk}{(2\pi)^D}\frac{-i}{k^2 + i\epsilon}\frac{1}{(k.n)(k.n^*) + i\xi}$$
$$\times[-2 + 2e^{ik.n} + 2e^{ik.n^*} - e^{ik.(n+n^*)} - e^{ik.(n-n^*)}] + I_R \ , \tag{18}$$

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where

$$I_R = i\pi \int \frac{d^D k}{(2\pi)^D} \frac{-i}{k^2 + i\epsilon} \delta[(k_0.n_0)^2 - (\mathbf{k.n})^2]$$
$$\times [-2 + 2e^{ik.n} + 2e^{ik.n^*} - e^{ik.(n+n^*)} - e^{ik.(n-n^*)}] \, , \tag{19}$$

which also vanishes by symmetric integration.

Consequently, we obtain for the Wilson loop (14) exactly the same expression as the corresponding in reference [8]:

$$W^{(1)} = (ig^2) C_F \mu^{4-D} 2n.n^* \int \frac{d^D k}{(2\pi)^D} \frac{-i}{k^2 + i\epsilon} \frac{1}{(k.n)(k.n^*) + i\xi}$$
$$\times [-2 + 2e^{ik.n} + 2e^{ik.n^*} - e^{ik.(n+n^*)} - e^{ik.(n-n^*)}]$$
$$= (ig^2) C_F \mu^{4-D} \frac{4\pi^{D/2}}{(2\pi)^D} i^{2-D/2} \int_0^1 dy \, (1-y)^{1-D/2}$$
$$\times \int_0^\infty dx \frac{e^{-x[(1-y)\epsilon + y\xi/n_0^2]}}{x^{D/2-1}} \left[2 - \exp(\frac{-in_0^2}{x}) - \exp(\frac{i\mathbf{n}^2}{x}) \right] \, . \tag{20}$$

Evaluation of (20) yields

$$W^{(1)} = \frac{g^2 C_F \mu^{4-D}}{(2\pi)^{D/2}} \frac{4\Gamma(D/2 - 1)}{(4-D)^2} [(n_0^2 + i\eta)^{2-D/2}$$
$$+ (-n_0^2 + i\eta)^{2-D/2} - 2(i\eta)^{2-D/2}] \, , \ \eta \to 0^+ \, . \tag{21}$$

The last term in square brackets in the above equation is absent in the corresponding expression of reference [8]. However, this is of no significance since we are considering the analytic extension of $W^{(1)}$ in the strip $3 < Re\, D < 4$ and, therefore, may set η equal to zero before making the expansion around $D = 4 - \epsilon$.

From the quoted results we may conclude that the unified-gauge formalism and the causal principal-value prescription are equivalent approaches to the Wilson loop in second order of perturbation theory for general axial-type gauges. A similar result was found regarding the Mandelstam-Leibbrandt (ML) and the ordinary PV prescription[9] for particular gauge choices.

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